

EECS C145B / BioE C165 Spring 2004:
Problem Set II (Revised)
Due February 27 2004

Please read the sections describing the rules for working in groups and the grading policy in the course introduction handout. Show all code and plots.

Goal: The purpose of this problem set is to gain an in-depth understanding about what points in the Fourier domain represent in real space (understanding the Fourier synthesis equation). The secondary purpose is to develop Matlab programming skills (sorting matrix elements, coordinate grids, loop structures, DFT representation).

Problem 1 (100 points)

1. Write a Matlab that generates the samples of a 2D sinusoid of arbitrary horizontal and vertical frequencies and phase and over a given Cartesian grid. The function might be called as:

```
function cosGrid = cos2d(u,v,phi, X, Y)
```

where X and Y have been generated by *meshgrid*.

2. Evaluate this function on a 160 column by 120 row grid that covers a width of 1 distance unit in real space. Add two cosines, one with a frequency of 0 in both directions and one with $u_0 = 4$, $v_0 = -1.3333$ and $\phi_0 = -\pi/4$.
3. Find the magnitude and phases at the highest and second highest spectral peaks.
4. We wish to use the DFT to look at the image as an expansion in terms of its Fourier components. By adding up the contributions of all the sinusoids that make up the image, we can see how an image may be regarded as the interference pattern created by summing many sinusoids of different frequencies, directions and phases. Each conjugate

symmetric pair of points in the Fourier domain represents one of these sinusoids. Formally:

$$f[m, n] = \frac{\kappa}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{A-1} |F[k, l]| \times \cos(2\pi(km/M + ln/N) + \phi[k, l])$$

where $A = \lfloor N/2 \rfloor + 1$ and the scale factor:

$$\begin{aligned} \kappa &= 1 && \text{for } k \geq B \text{ and } l = 0 \\ \kappa &= 1 && \text{for } k = M/2, M \text{ even or } l = N/2 = (A - 1), \\ &&& N \text{ even} \\ \kappa &= 2 && \text{otherwise} \end{aligned}$$

where $B = \lfloor M/2 \rfloor + 1$. This sum is carried out on a single half-space of the Fourier domain because of the conjugate symmetry that occurs in the transform of a real image. This allows us to make up the expansion from real sinusoidal function rather than complex exponentials.

To do this:

- (a) Evaluate the magnitude and phase of the DFT of the image.
- (b) Shift these so that DC is at the center.
- (c) Make a Cartesian grid of horizontal and vertical frequencies for these matrices. After shifting, the vertical axis runs up the rows of the matrix from $-v_s/2$ up to $v_s/2 - v_s/N$ in steps of v_s/N . The horizontal axis runs across the columns of the matrix from $-u_s/2$ up to $u_s/2 - u_s/M$ in steps of u_s/M .
- (d) Now extract quadrants III and IV of the magnitude, phase and coordinate matrices by selecting the first A rows. Check that the bottom row of the extracted half-space magnitude contains the DC component at its center. We extract these quadrants since these contain the highest absolute frequency components at $v = -v_s/2$ while quadrants I and II do not. Because the image is real, there is no theoretical difference in extracting the upper or lower half-space.
- (e) Convert the extracted half-space magnitude matrix to a vector. Use the *sort* function to sort this vector in ascending order and return the matrix indices of the ascending values.
- (f) Loop backwards through these indices (use a *for* loop). Retrieve the frequency, magnitude and phase information for each in turn. Use your cosine generating code to create each sinusoid that corresponds to each of these spectral values. In this way, you will be adding the sinusoids that contribute to the image in decreasing order of their power contribution to the image. After a sinusoid with the correct parameters has been generated, add it to a running sum image of the components. As the loop advances, the sum

image should start looking more and more like your original image. The first component should have $u = 0$ and $v = 0$ and have an amplitude close to 1. The second should have $u = 4$, $v = -1.3333$ and also have a magnitude close to 1. The *pause* command will be helpful in debugging your code. At every step, plot the current sinusoidal component, the location of the corresponding point on the frequency plane, and the running sum image.

- (g) Download the image at:

`http://muti.lbl.gov/145b/images/retroanon.jpg`

Either use a web browser to do this, or on some Unix systems you can simply type:

`wget http://muti.lbl.gov/145b/images/retroanon.jpg`

- (h) Using *imresize*, reduce the resolution of the image by a factor of 8.
- (i) Apply your Fourier synthesis program to this image. Note how the image starts to form out of the interference pattern. Print out the final running sum image and the original image.
- (j) Qualitatively describe the nature of the sequence of the components in terms of how these contribute to image features.
- (k) Plot a graph showing the decrease in the magnitude of each successive component.

Your program should give output similar to this when applied to 1st image:

